

Geometric Unification of Classical Gravitational and Electromagnetic Interaction in Five Dimensions: a Modified Approach

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A unification of the gravitational with the electromagnetic interaction within a classical framework is proposed. It is based on a V_5 -geometry, with $x_5 = q/m$. The sole source term is mechanical stress energy, positioned along x_5 . The trajectories of testbodies are placed in V_4 ($x_5 = \text{const}$)-slices. The final field equation couples a geometric G -tensor to mechanical stress energy, its momentum with respect to x_5 and the change of this momentum with proper time τ .

Introduction

Apart from the successful unification of the electric and the magnetic forces themselves, the two fundamental interactions out of the remaining four attracting the most and most prolonged attention for unification attempts, are the gravitational and the electromagnetic ones. Pre-relativity phenomenology presented very little scope for such attempts, and only the description of gravitation by the theory of general relativity seemed to offer a promising basis: the geometric replacement of natural forces by manifold curvature. Thus, already rather soon after the introduction of general relativity, first unification possibilities were explored, and this continued practically up to the present day. Einstein himself spent the main part of his later working life, searching for a real unification between gravitation and electromagnetism. In the more recent past, tendencies have shifted with the advent of a relatively successful introduction of group theory calculus into elementary particle physics. Present day interest centres around the unification of electromagnetic and weak interaction (plus perhaps the strong force), but the group theoretical tools seem to defy, for the time being, the integration of gravitational phenomena.

There is no need to go into detail with regard to the individual contributions to the unification attempts between gravitation and electromagnetism (our main concern here). A variety of excellent reviews exists, being the result of or again a source for others [1, 2]. One can distinguish basically two lines of approach: the affine connection method in

four dimensions (space-time manifold) and the five dimensional manifold with its additional dimension being the ratio q/m (charge/mass).

The majority of present day theoreticians consider all past attempts as having been unsuccessful. This is certainly true for those yielding e.g. charged particle trajectories from the geodesic equation, which have no resemblance to reality (e.g. Einstein's affine connection theory). But other, more successful approaches are equally rejected on the grounds that either their unification emanates into already known physical structures and therefore renders itself meaningless (Weyl; his unified theory resulted in the established Maxwellian equations and therefore presented nothing excitingly new), or that the achieved unification was not a true one. Most of the five-dimensional theories are dismissed for the latter reason. It is argued, that a 5th dimension is physically unobservable. Another criterium is the non-total elimination of a characteristic electromagnetic part of the stress energy tensor, which is practically true for any of the past attempts. However, the rejection arguments are nowadays open to debate again. Some authors [3] argue that a true unification has already been achieved by the incorporation of electromagnetic terms into the stress energy tensor in Einstein's field equation; on the other hand the $(4+n)$ -dimensionality concept sees a revival in gauge theoretical unification attempts of the "electroweak" with the strong force.

This paper presents a new approach to the unification of the gravitational with the electromagnetic interaction on a geometrical basis in 5 dimensions. Initially, only classical phenomena are considered. The criteria to be met for a "true" unification are defined. By taking these criteria and an elevated

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“principle of equivalence” as theoretical basis and a V_5 -manifold with $x_5 = q/m$ as tool, one is lead to a qualitative, heuristic outline of the whole unification concept. After its presentation the formulative results are laid down in a separate paragraph with an interpretation of the new resulting field equation. Finally, the classical limits of the theory will be touched, speculating on application on quantum systems and progressive unification with the remaining forces in nature.

Heuristic Concept

The criteria for a true unification between gravitation and electromagnetism are the following:

(a) Both interactions have to be described by the same field equation, relating all field effects to the same source terms. If the unification is placed on a geometric basis, one and the same geometric structure must serve as a medium for the description of all these field effects.

(b) On a geometric basis all forces have to be replaced by a coupling of a single type main source term to geometry. No special remaining terms relating to one or the other non-unified descriptions are permitted.

(c) All dimensions of an n -dimensional manifold, presenting the geometrical framework, must be in some way physically observable. There are two principal lines of approach conceivable: a phenomenological one, based on geometry, and a more axiomatic one—both lead to the same consequences. Let us first consider the phenomenological one.

The concept of force itself seems to be at the root of the present day split into four different types of interaction. Therefore to abandon it and place the description of physical effects on directly observable quantities, i. e. dimensions and energy, is a precondition for unification. This is exactly, what general relativity does: the gravitational force is replaced by the coupling of curvature of a space-time manifold to an energy source term. It is therefore likely that only geometrical attempts for unification will be successful. As a consequence, the electromagnetic “force” as well has to be replaced by a coupling of a manifold curvature to an energy source term. Both the manifold, in which the action takes place, and the source have to be common to gravitational and electromagnetic events. This excludes

immediately the V_4 -geometry of general relativity as a unification basis and indicates for that at least a five-dimensional manifold. The common source has to be energy (related), bare of any specific terms with reference to one or the other interaction type.

The more axiomatic approach is outlined as follows: Let us postulate an “elevated principle of equivalence”: “If an observer in a (special) inertial frame (i. e. a Faraday cage) is travelling in an ‘accelerating’ field, he can neither distinguish, whether he is moving in a gravitational or an electromagnetic or any other field, nor whether he is subjected to an apparent field (of any kind) due to acceleration of a reference frame.”

This principle is a sufficient basis, to arrive with at the same conclusions, as the phenomenological approach did.

The following model in a V_5 -manifold offers itself: We have three space-like, one time-like dimension and x_5 with (q/m) as affine parameter. Testbodies and energy sources (masses, etc.) reside along x_5 according to their (q/m) -value in four-dimensional slices V_4 ($x_5 = \text{const}$). Each V_4 has its own space-time geometry. Let us for simplicity assume that a testbody or a source does never change position with respect to x_5 (In $V_4(x_5 \equiv 0)$ only “neutral” test bodies move). For one and the same source term at a specific x_5 the strength of the curvatures of the various V_4 ’s is normally different. Thus the V_4 ’s may appear “blown-up” or “contracted” with reference to each other and probed by the trajectories of testparticles, due to the gross curvature of the whole V_5 . Equally the gross curvature of the V_5 changes, and thus the relative blow-up or contraction of the V_4 ’s to each other, when the source is positioned at a different x_5 . There is no real difference between a testbody and a source; their roles can be played by one and the same entity, and in mutual interactions of several testbodies they are both at the same time. Thus we can postulate:

The curvature of the entire V_5 geometry is dependent on the stress energy of a source *and* the position of the source along x_5 .

This is not quite all. As will be seen in the quantitative results, curvature may also be coupled to the velocity of certain types of sources. The Table gives possible combinations testbody/source.

For the illustration of the above, let us pick the most general example, where both testbody and

Table 1. Terminology.

Old		New	
testbody	source	testbody	source
neutral	neutral	in $V_4 (x_5 = 0)$	in $V_4 (x_5 = 0)$
neutral	charged	in $V_4 (x_5 = 0)$	in $V_4 (x_5 \neq 0)$
charged	charged	in $V_4 (x_5 \neq 0)$	in $V_4 (x_5 \neq 0)$
charged	neutral	in $V_4 (x_5 \neq 0)$	in $V_4 (x_5 = 0)$

source are “charged”. The source A resides at $x_5 = a$. Thus the entire V_5 is curved initially with respect to the strength $|A|$ and the position value a . If A would reside at $x_5 = b \neq a$ the gross curvature of the V_5 would be different. A testbody C moves in a $V_4 (x_5 = c)$. In general $c \neq a$. The trajectory of C is dependent on the curvature of $V_4 (c)$. A testbody D at $x_5 = d \neq c (\neq a)$ would move in a different curved V_4 and thus project a different trajectory.

In traditional language the above means: the trajectory of a testbody with mass m_{t1} and charge q_{t1} under the influence of a source with mass m_{s1} and charge q_{s1} changes, when the source is replaced by one with $m_{s2} \neq m_{s1}$ and $q_{s2} \neq q_{s1}$; or: the trajectory of a testbody with m_{t1} and q_{t1} under the influence of a source with m_{s1} and q_{s1} is different from that of a testbody with $m_{t2} \neq m_{t1}$ and $q_{t2} \neq q_{t1}$ under the influence of the same source.

How does the general structure of a V_5 look like? Figure 1 illustrates that a five-dimensional universe is a flat sheet, or rather an assembly of an infinite number of V_4 sheets piled onto each other. It is not infinite in the x_5 -direction, since the maximum observed and for the time being theoretically possible value for x_5 is (e/m_e) , the ratio of the electron's charge over its mass. Therefore the regions I and II in Fig. 1 are physically meaningless.

Figure 2 shows space-time curves (particle trajectories e. g.) for constant time. The V_5 , subjected to the influence of a source, somewhere situated

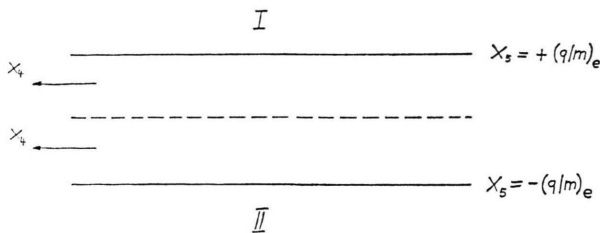
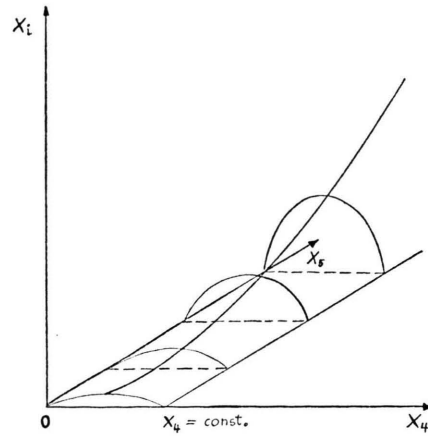
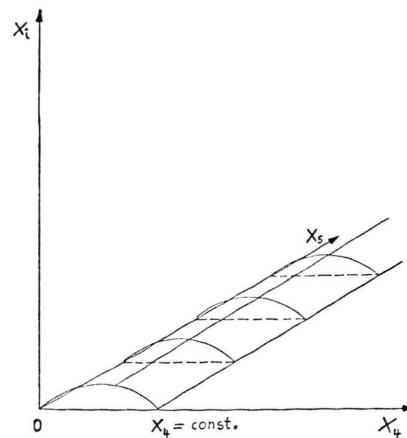


Fig. 1. Global 5-Dimensional Universes.

Fig. 2. Particle trajectories for constant time in a curved V_5 -manifold.

along x_5 , accomodates testparticles moving in various V_4 planes. The gross curvature of the V_5 determines the “blowing-up” of trajectories in the V_4 's along x_5 . Thus, each geometry is coupled to one and the same source exactly as in general relativity, only the coupling constants and the weight of the x_5 -position change along x_5 .

Figure 3 depicts the case $q_s = 0$. If a source is situated at $x_5 = 0$ the V_5 is flat and all V_4 's are curved in parallel (only “gravitation”). If a testbody is placed at $x_5 = 0$ it will always move on a geodesic, entirely determined by the strength of the source alone, independent of the position of the source and almost negligible with respect to the adjacent ones along x_5 as “rest-curvature”.

Fig. 3. Particle trajectories in a flat V_5 .

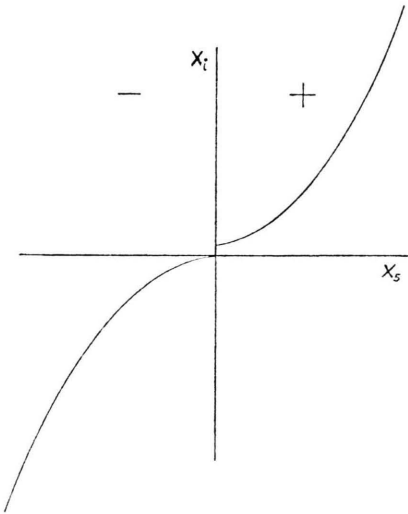


Fig. 4. Positive and negative sectors of the V_5 -manifold under curvature.

As was already obvious from Fig. 1, the V_5 -univers is divided into two sectors, a positive and a negative (with respect to x_5 assignment). If a source curves the positive sector a certain way, this curvature will be repeated inversly in the negative sector (repulsion and attraction) (s. Figure 4).

What has been achieved so far? The model eliminates the concept of charge, by postulating that testbodies and sources posses only mass or energy, positioned along a fifth dimension at various locations (charge eliminated by geometric concept). The electromagnetic force has been eliminated by coupling of the curvature of a five-dimensional manifold to "neutral" strength and position of such a source (always "neutral", only stress energy).

The unification criteria will be revisited after the quantitative presentation in the following.

Quantitative Results

The mathematically tedious construction of the new unified field equation shall not be outlined in detail here. Three main points have been observed:

- (i) the quantitative presentation of the theory has to follow rigourously the heuristic expectations;
- (ii) it has to result in Einstein's field equation for the case $x_5 = 0$;
- (iii) the "electromagnetic" part of the stress energy tensor had to be reduced to its "mechanical" parts only, by extracting the charge and incorporating it into x_5 .

Thus the new unified field equation can be expressed in the following way (without explicit indices) :

$$G = \left(\kappa_1 + \kappa_2 x_5 + u \kappa_3 x_5 \frac{\partial}{\partial \tau} \right) T,$$

T is the purely mechanical stress energy, u the 4-velocity of a test body, $\partial \tau$ an interval of proper time,

$$\kappa_1 = 8 \pi,$$

$$\kappa_2 = 1 / (A_1 \epsilon_r \epsilon_0),$$

$$\kappa_3 = \frac{\mu_r \mu_0}{4 \pi A_2} \int \frac{1}{r^2} \sin \gamma \, dS,$$

with the various terms in κ_2 and κ_3 resulting from the old electromagnetic stress energy part.

G is the curvature tensor of the V_5 manifold. It can be constructed by the usual well known relations from general relativity, i. e., the Riemannian tensor for V_5 , the corresponding connection coefficients and the metric

$$\Delta s = (g_{\mu\nu} \Delta x^\mu \Delta x^\nu)^{1/2},$$

with

$$x = x_i \text{ and } i = 1, 2, 3, 4, 5$$

$$x_4 \text{ time-like, } x_5 \text{ "charge"-like } (q/m),$$

$$15 \text{ metric coefficients } g_{\mu\nu}.$$

For the construction of G and the derivation of the geodesic equation one can either start from the global V_5 and end up at the various or particular V_4 's or from the V_4 's and build up the global V_5 geometry. Both approaches, which are complementary, may make use of a method, developed by Arnowitt, Deser and Misner [4], briefly ADM-method, which connects a V_n to a V_{n-1} . For the Riemann tensor this looks like (with explicit indices) :

$${}^{(n)}R_{ijk}^m = {}^{(n-1)}R_{ijk}^m + (n-1) {}^{(n-1)}(K_{ij} K_k^m - K_{ik} K_j^m),$$

with

$$K_{im} = n \frac{\partial}{\partial x^i} e_m.$$

K is called the extrinsic curvature operator, n a normal vector, dn would be the vectorial difference, generated by K , after transporting n parallel within a hypersurface. Thus K relates the intrinsic curvature in a V_{n-1} to the global extrinsic curvature of the V_n itself;

$$\frac{\partial}{\partial x^i} e_m = {}^{(n)}\Gamma_{mi}^\mu e_\mu, \text{ } e_\mu \text{ being a basis.}$$

We can interpret the unified field equation as follows: The curvature of a V_5 -manifold under the action of a unified gravitational/electromagnetic field depends on a mechanical stress energy tensor T , i. e. on its absolute value, its momentum with respect to x_5 and the change of this momentum with proper time.

This is a somewhat unexpected result and differs entirely in shape from the purely gravitational Einstein equation. However, some assumptions and simplifications have been introduced along the lines of the development of the equation and these are briefly recalled here.

T was assumed to be fixed along x_5 , therefore $\partial/\partial\tau$ has to be applied to T , rendering the source part dependent on a stress energy flux, corresponding when multiplied by x_5 to the classical magnetic forces arising from the movement of charges in traditional descriptions. If T is assumed to be changing along x_5 , one would always have to consider $\partial(x_5 T)/\partial\tau$. This is e. g. important in ionisation processes and mass conversion. In the first case the affine parameter in a geodesic equation is most likely x_4 , resulting in a trajectory of a testbody as a geodesic in a V_4 . In the second case x_5 could be affine parameter as well, resulting in the trajectory of a testbody changing its charge or mass during the interval under consideration.

The other assumption was that κ_2 and κ_3 were to be considered uniform in any of the directions. This may be true for most cases, in general, however, they should be presented as indexed tensors, their components differing according to possible anisotropies within a specific coordinate frame.

Discussion and Conclusions

Summarising, one can say that a formalism has been proposed to describe the nature of gravitational and electromagnetic fields in a unified way. The resulting field-equation reduces to the Einsteinian in the absence of an electromagnetic field, thus enlarging the context of general relativity. The basis of the unification is a V_5 -geometry coupled to a mechanical stress energy tensor, positioned along an x_5 -dimension. The affine parameter of the 5th dimension is the ratio (q/m) , charge/mass.

Let us go back to the criteria for true unification.

(a) Evidently the first condition is fulfilled: one field equation, a common source term, one and the same geometric structure.

(b) Only the mechanical stress energy remains as source; electromagnetic contributions have been accounted for by the type of coupling to geometry, the stress energy momentum along x_5 and the change with proper time of the latter.

(c) One could argue, that x_5 is physically unobservable. If we forget its derivation from electromagnetics, however, and regard it as true geometric dimension, it is observable, when going backwards from the field equation: the position, along which a purely mechanical source is situated in this dimension, is observable by interpretation of the geodesics of testbodies under the influence of such a source. Inversely, when we know the position of a source, we can predict the movement of testbodies at a specific position along x_5 . Thus the "depth" of x_5 can be probed, i. e. observed.

A unification of two out of the four known forces in nature should really only make sense with a view on further unification attempts with the remaining interactions. This does not mean, that it is necessarily possible, but behind it stays the same metaphysical drive that lead to a unification of the first two in the first place. Further unification now has to proceed along the same lines as above: on a geometric basis. But it would have to include quantum theoretical elements, since the strong and weak interactions are closely bound to quantum systems. Before one would be able to proceed further, the present field equation would have to be refined to include at least two features relating to quantum theory: spin and the quantization of geometry itself. The latter is perhaps only required for the strong and weak interaction, but certain aspects of it and certainly spin may already play an important role in the quantum mechanical application of the unified field equation, since at least electromagnetism has its domain in quantum systems as well. In this context, it is interesting to remark, that the concept of geodesic would have to be redefined radically, if ever a geometric description of quantum systems became possible — a geodesic would have to become a probabilistic world line, something rather contradictory to the determinism of general relativity. This, however, takes us already to the limits of the present paper, which was supposed to treat only classical systems.

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